EXPERIMENTS WITH NON-LINEAR FILTERS
Discovering Excitable Regions

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Abstract

We describe the development and implementation of a family of non-linear filters in which the non-linear term is given a variable recursive delay, in the context of a search for what we term an *arithmetical instrument* – an instrument whose design is founded on purely numerical models.

Such filters are inherently unstable; we describe how they may be controlled, not only by the careful selection and constraining of parameters, but also by appropriate and idiomatic performance technique. We draw comparisons between these filters and the behaviour of acoustic instruments, which typically exhibit what we call *excitable regions* of sonic activity.

1 Introduction

Non-linear techniques are widely used in sound synthesis, in the forms of a variety of modulation techniques, among the most significant of which have been FM synthesis [Chowning, 1973] and non-linear waveshaping [Arfib, 1979][Beauchamp, 1979][LeBrun, 1979]. The term also applies to standard signal-processing functions such as gating and compression. In these techniques, the non-linear behaviour is strictly controlled, and is applied by means of linear arithmetical processes. In the case of waveshaping and compression, for example, the non-linearity applies to a fixed transfer function to which incoming samples are mapped – the mapping of input values to output values is itself a simple linear process.

In general, any signal processing algorithm which is either time or input dependent may be classed as a non-linear system [Reid and Passin, 1992]. Such behaviour is normally avoided in digital filter algorithms because of the well-known problems of stability [Lynn and Fuerst, 1989]. They do however find a place in non-musical applications such as image processing [Embree and Kimble, 1991].

Our investigations of non-linear filters arise out of a quest for an *arithmetical instrument* – an instrument which is not drawn from a physical model but which is defined entirely by numbers. There are two primary aspects to any instrument – the sound generator mechanism, and its environment. In the physical world, this would apply, for example, to a set of strings, and to the violin body on which they are mounted. Here the primary non-linear element is the string: up to a point, increasing its tension will increase its fundamental rate of vibration; beyond that point it will break. Similarly, below a minimum tension it will not vibrate at all. We may therefore describe the string as having an *excitable region*, only within which is it useful musically. The instrument as a whole has four such regions; it is not possible to perform a continuous glissando from the lowest to the highest available notes.

The problem with conventional synthesis techniques (from our point of view) is that they do not in general exhibit such bounded excitable regions – a linear oscillator may easily be designed to span and exceed the human audible range. Only with the development of techniques of physical modelling drawing on aspects of chaos theory [Rodet, 1994][Mackenzie, 1995] has the essentially non-linear behaviour of musical instruments (from which much of their expression derives) begun to be realistically captured in synthesis.

Just as an instrumentalist first accepts, and then strives to exploit and transcend the so-called limitations of their instrument, so we in our researches are relatively unconcerned to use mathematical artifices to overcome all analogous limitations in a given filter; rather, we see such limitations as features of an instrument which an idiomatic performance technique can exploit musically. We give one example of this below.

2 The Model

In our model, the sound source is provided by a non-linear oscillator algorithm [Dobson and Fitch, 1995], adapted from the simple recurrence relation of the Mandelbrot set [Mandelbrot, 1992]:

$$X_{new} = X_{old}^2 - C \qquad (1)$$

This can be shown to be stable for values of $0 < C < 2$.

In extending this formula to create musically useful waveforms we have added one or more delay elements, leading to a recurrence relation which combines a conventional linear filter with a non-linear delay:

$$X_n = a.X_{n-1} + bX_{n-2} + cX_{n-L} + C \qquad (2)$$

We have found that through careful selection of coefficient values, a wide range of decaying and sustained sounds, often of richly time-varying spectral character, can be realized. Above all, these oscillators exhibit *Excitable Regions*, outside which they will either converge rapidly, or exceed the bounds of their numerical representation (in effect, they will break). Within these regions, they can be forced into unexpected modes of oscillation, in much the same way that a wind instrument will generate multiphonics or squeak, if played incorrectly or unconventionally.

We have, therefore, the first element of our arithmetical instrument. We now turn to the second element, the environment or body, which we also wish to generate through non-linear means. While it is possible to take a sampled impulse response and convolve it with the input signal (for example, using the new convolve unit recently added to Csound [Vercoe, 1993], we have eschewed this ‘safe’ method in favour of a direct algorithmic implementation.

Returning to our oscillator (2), we recast this to receive an input X_n:

$$Y_n = aY_{n-1} + bY_{n-2} + dY^2_{n-L} + X_n - C \qquad (3)$$

This can be described as a conventional linear Infinite Impulse Response (IIR) filter combined with a non-linear delay length L, plus the constant term. We have chosen to focus on the non-linear aspect; accordingly we disregard the linear terms for the moment:

$$Y_n = dY^2_{n-L} - C \qquad (4)$$

where, for stability, $0 < c \leq 1, 0 < d < 1$ (*i.e.* d and C are opposite in sign, but may be of similar magnitude, $|C|$ usually less than $|d|$)

The constant term C has the unavoidable side-effect of imposing a DC component, with starting transient, on the output. Although the filter exhibits useful behaviour without it, we have found that the most striking effects are dependent on its inclusion, despite the concomitant need for a DC removal filter on the output.

Although the filter is by its nature sensitive to the input, we have so far had no problems with stability so long as the input is constrained within the range $-0.5 < X_n < 0.5$. This is however a maximum; the chaotic behaviour develops rapidly as the input rises above -12dB.

It is impossible to illustrate all the distinctive behaviours of this filter; a fairly representative selection of results in the frequency domain is shown in (Fig.1); in each case the spectrum of the output given a chirp input is shown. We have given examples which fill a large part of the spectrum, so that the filters’ behaviour can be seen clearly; practical filters would concentrate more of the activity in the lower frequency ranges.

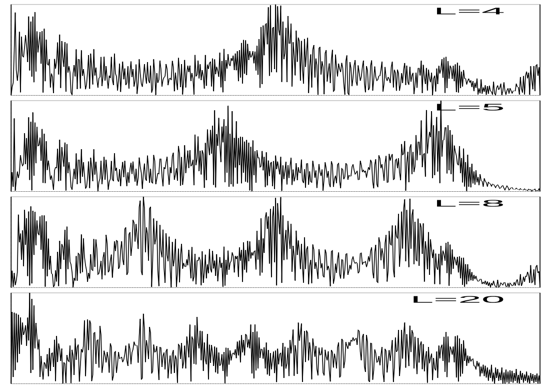

Figure 1: Selection of spectra $C = 0.4, d = 0.42$

It will be seen that the filter exhibits distinctly comb-like behaviour as the delay L is increased. However, unlike the response of a linear comb filter, in which the peaks are regular in spacing, width and amplitude, this filter exhibits marked irregularities in all those aspects. Some responses look remarkably like formant regions, indicating that this filter has a distinctive set of personalities, and is therefore able to provide a range of ‘bodies’ for our arithmetical instrument.

However, we consider the single most significant property of this filter to be the fact that, whereas an input sine wave will excite but one resonant peak of a linear comb filter, it will excite several in this case; the filter’s Excitable Regions will be multiply activated in a moderately unpredictable way according to the characteristics of the input. Accordingly, we call this and its derivatives an *ER filter*.

We now consider briefly the inclusion of the linear filter elements according to formula (3). On their own they form a simple resonator (band-pass), which can be configured also as a low or high-pass filter. (In a practical implementation, the coefficients a and b may be calculated conventionally, for a given centre frequency and sample rate.)

The combination of the linear and non-linear elements can be interpreted in two equally useful ways.

Firstly, d and C can be taken together as a kind of modulation or ‘fractal’ index (whose range must clearly reduced according to the gain of the linear filter), which, as it increases from zero, adds a comb-like irregularity to the linear band-pass response – the ER filter can be ‘fractalized’ dynamically. For stability the range of the index must be reduced according to the gain of the linear filter; putting it another way, we want to leave room for non-linear behaviour to be introduced by keeping the linear gain within conservative levels.

We note that in this way (by always commencing the fractal index from zero) the startup transient described above can be obviated – this is therefore an idiomatic performance technique for this filter. Appropriately for this interpretation, the delay L can be set at 3 for the clos-

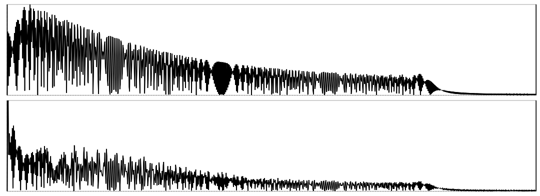

Figure 2: Linear and Chaotic with low offset

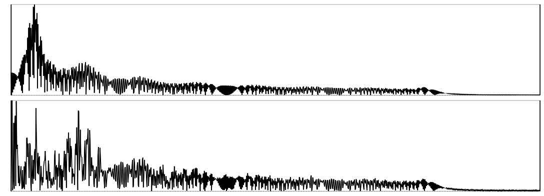

Figure 3: Linear and Chaotic Recursive Comb

est match between chaotic and linear responses (Fig.2), or it can be set to any reasonable distance as for a comb filter.

Alternatively, the formula can be regarded as a non-linear comb filter with (typically) a low pass or low-frequency band-pass linear filter on the output, serving primarily to remove those extreme high-frequency components which will inevitably arise from the non-linear behaviour.

This further suggests a possible application of the ER filter as the basis of a non-linear reverberator. Our experiments with this are still at an early stage; we are not expecting to invite any qualitative comparisons with normal reverb (although with the interest in some commercial quarters for 'retro' sounds such as plate and even spring reverbs we do not dismiss the thought entirely), rather we see this as a means to enhance the sense of the 'size' or 'presence' of our arithmetical instrument in a manner consistent with our overall approach.

In our previous investigations of non-linear oscillators [Dobson and Fitch, 1995], we have found that the application of a delay offset to the second linear element led to a new family of musically worthwhile sounds, especially when the offset is varied dynamically. Applying the same approach to the ER filter leads to the formula

$$Y_n = aY_{n-1} + bY_{n-M} + dY_{n-L} + X_n - C$$

where $2 < M < L$

In the absence of the non-linear components this amounts to a linear recursive comb filter, which though unorthodox (the standard comb filter includes a carefully matched non-recursive delay) seems worthy of a deeper investigation in itself. In our present context we have noted that introduction of the non-linear components does, somewhat to our surprise, substantially preserve the linear frequency response (Fig.3). The offset M is anti-symmetric in character; given $L = 100$, for example, the response where $M = 10$ is the inverse of the response where $M = 90$ (Fig 4).

3 Conclusion

As we have found with respect to our non-linear oscillators, the ER filter exhibits a wide variety of musically useful behaviours, any one of which invites deeper exploration. We have found it to be unexpectedly easy to control, and see it as highly amenable to a real-time implementation on a DSP. It is also simple to implement in CSound, which (especially if it can be run in real-time), would appear to be an ideal tool with which to explore both the ER filter and the non-linear oscillators to which it is so closed related.

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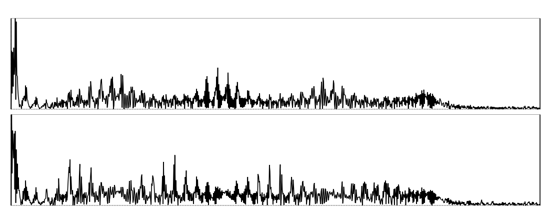

Figure 4: ER filter with linear offset 10 and 90

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